DO NOW

Where do vertical asymptotes exist on a function?

2.5 Infinite Limits

An infinite limit is: a limit in which $f(x)$ increases or decreases without bound as $x \to c$.

Notation: $\lim_{x \to c} = -\infty$ OR $\lim_{x \to c} = \infty$

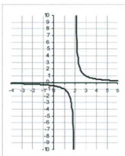
Be sure to understand that: this does not mean that the limit exists. It tells you how the limit fails to exist by denoting the unbounded behavior of $f(x)$ as $x \to c$.

*Remember for a limit to exist it must be a real number.

Vertical Asymptotes:

For $h(x) = \frac{f(x)}{g(x)}$, such that $f(c) \neq 0$ and $g(c) = 0$ for all $x = c$, then $x = c$ is a vertical asymptote of $h(x)$

*$h(x)$ approaches infinity as x apprachës c from the left or right.



Ex: Vertical asymptote at $x = 2$

To find vertical asymptotes:

1. Factor completely and cancel (*removable discontinuity)
2. Remaining factors in denominator are where vertical asymptotes occur. (nonremovable discontinuity)

Also FYI: Remaining factors in numerator are where x-intercepts occur.

***Be sure to check:

left and right limits

Theorem 2.15 Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions such that:

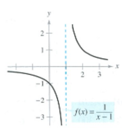
$\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$

1. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty$
2. Product: $\lim_{x \to c} [f(x)g(x)] = \infty$
3. Quotient: $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$

** Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$

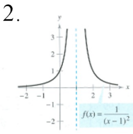
Examples:

Determine the limit of each function as x approaches 1 from the left and from the right.

1. 

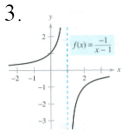
$\lim_{x \to 1^-} = -\infty$

$\lim_{x \to 1^+} = \infty$

2. 

$\lim_{x \to 1^-} = \infty$

$\lim_{x \to 1^+} = \infty$

3. 

$\lim_{x \to 1^-} = \infty$

$\lim_{x \to 1^+} = -\infty$

Examples: Find the vertical asymptotes (if any) of the function.

1. $f(x) = \frac{2}{x - 4}$

$x - 4 = 0$

$x = 4$

2. $f(x) = \frac{x^2 + 2x - 8}{x^2 - 4}$

$\frac{(x+4)(x-2)}{(x+2)(x-2)}$

$x = -2$

Page 7

Find the limit.

3. $\lim_{x \to 4^+} \frac{2}{x - 4} = \infty$

$f(4.1) = 20$
$f(4.01) = 200$
$f(4.001) = 2000$

4. $\lim_{x \to 4^-} \frac{2}{x - 4} = -\infty$

$f(3.9) = -20$
$f(3.99) = -200$
$f(3.999) = -2000$

5. $\lim_{x \to -2^-} \frac{x^2 + 2x - 8}{x^2 - 4} = -\infty$

$f(-2.1) = -19$
$f(-2.01) = -199$
$f(-2.001) = -1999$

6. $\lim_{x \to -2^+} \frac{x^2 + 2x - 8}{x^2 - 4} = \infty$

$f(-1.9) = 21$
$f(-1.99) = 201$
$f(-1.999) = 2001$

Page 8

HOMEWORK

pg 108 - 109; 1 - 15 odd, 19 - 29 odd,
33, 35, 39 - 51 odd

Page 9